Growing Patterns
Function Tables and Rules – Evaluating Expressions
Coordinate Graphing

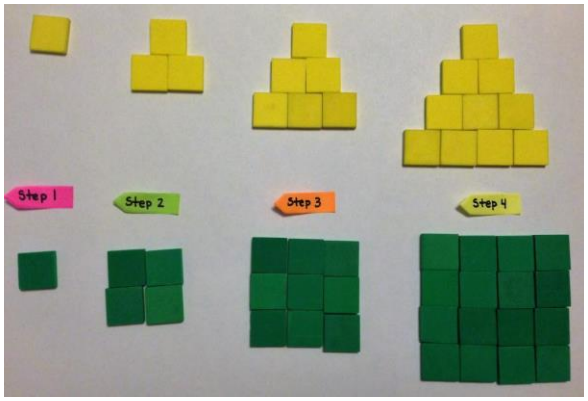
Growing patterns data can be recorded on function tables, as expressions with one variable, and as ordered pairs on an x/y coordinate graph. Students should be able to physically create or extend the growing pattern with concrete manipulatives and in pictorial form.

Types of growing patterns:

1. Arithmetic Sequence: This pattern grows (or shrinks) by the same amount with each successive number. It is additive or subtractive. Examples:
   - 2, 4, 6, 8, 10 (+2)
   - 3, 9, 15, 21, 27 (+6)
   - 5, 10, 15, 20, 25 (+5)
   - 4, 8, 12, 16, 20, 24 (+4)
   - 20, 17, 14, 11, 8 (-3)
   - 50, 40, 30, 20, 10 (-10)

2. Geometric Sequence: In these patterns, the previous number is multiplied by a constant ratio. Examples:
   - 2, 4, 8, 16, 32 (x2)
   - 2, 8, 32, 128 (x4)
   - 2, 6, 18, 54, 162 (x3)

3. Neither one: The change from one number to the next may grow instead of remain constant. Examples:
   - 1, 3, 6, 10, 15 (+2, +3, +4, +5, etc.)
   - 1, 2, 4, 9, 16 (These are all squared #)



These skip counting patterns can be recorded with:

- A function table. The left column would be the sequence # or step # and the right column would be the resulting term. A rule or formula would be shown.
- A recursive rule: This tells how to get from the first term to each successive term. The recursive rules are listed above for the arithmetic and geometric sequences. These are the patterns you see as you skip count or go down the right column in the table.
- An explicit formula: This formula can be applied to any step. If you wanted to know the 100^{th} term, you would not spend time to repeatedly add or multiply over and over again.

You would apply the explicit formula which shows the relationship between the step # and the result.

- Coordinate graph. Using the data in the function chart, the left column would be x, the right column would be y.
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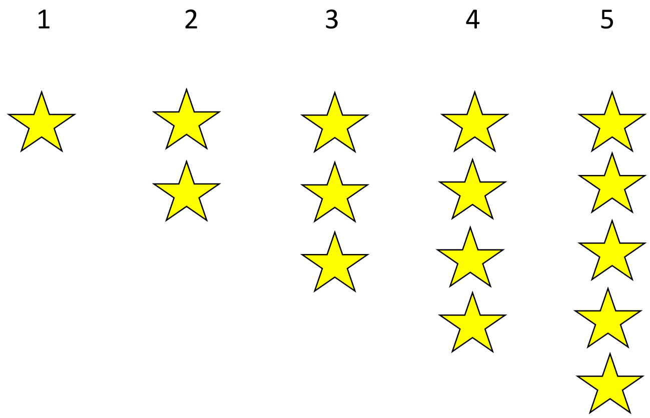
Arithmetic Sequence example: 5, 10, 15, 20 How many points on 1 star? On 2 stars? Etc.

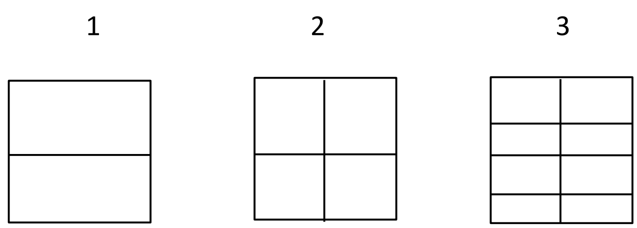
Rule: + 5 Formula: 5*n*	
Step # (n) (# of stars)	Result (# of points)
1	5
2	10
3	15
4	20
5	25



The **recursive rule** is (+5) because each result increases by 5. The **explicit formula** is expressed as an expression with a variable (5n). To solve for any number of stars (n), you multiply 5 times *n*. Example: For 8 stars, the explicit formula is 5 x 8 = 40 points.

Geometric sequence example: 2, 4, 8, 16, 32 Each time you fold a sheet of paper, your result is how many sections?

Rule: x2 Formula: $2\bullet 2^{(n-1)}$	
Step #(n) # of folds	Result (# of sections)
1	2
2	4
3	8
4	?
5	?



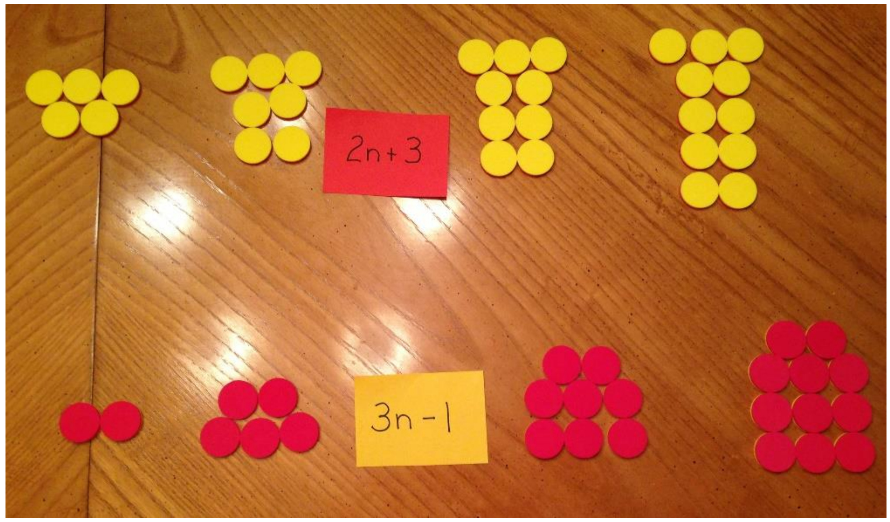
The **recursive rule** is (x2) because to get the next result, you multiply x 2 (going vertically down the result column). Since this is a geometric pattern, the **explicit formula** is more complicated and would be $2\bullet r^{(n-1)}$. Don't expect this formula knowledge for 5th or 6th graders.

The 2 is the first number in the sequence, and the r is the common ratio, which in this case is 2.
Example: To find the 5th term, the formula would be $2 \bullet 2^{(5-)}$ or 2 x 2 to the 4th or 2 x 16 = 32.
To find the 10th term, the formula would be $2 \bullet 2^{(10-1)}$ or 2 x 2 to the 9th or 2 x 512 = 1024

More examples of Arithmetic Sequence Patterns

Rule: +2 Formula: 2*n* + 3	
Step #	Result
1	5
2	7
3	9
4	11
5	?

Top picture

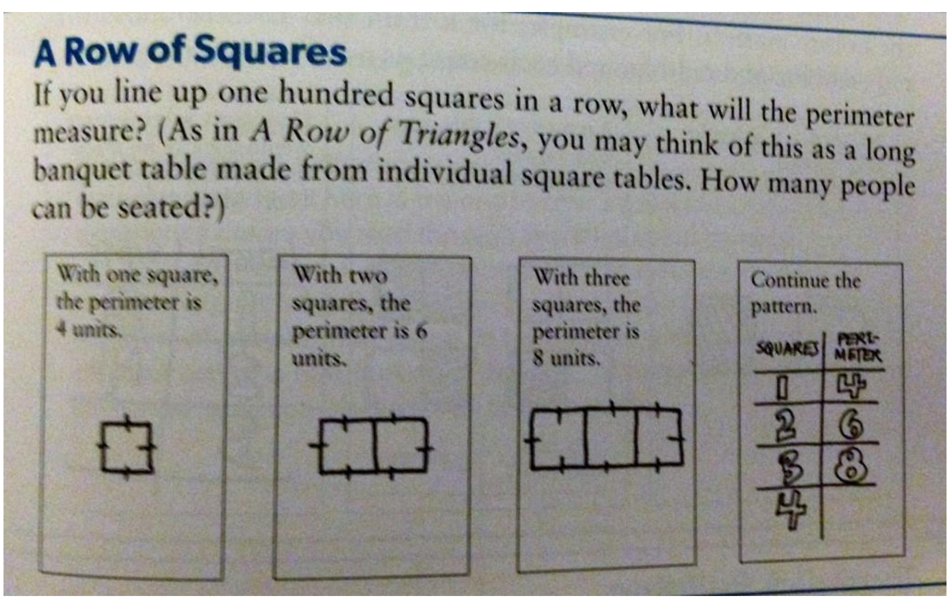


Rule: +3 Formula: 3*n* -1	
Step #	Result
1	2
2	5
3	8
4	11
5	?

Bottom picture (see coord. Graph on page 6)

There are many ways to illustrate this formula. These are just 2 examples.

The following growing pattern examples are from About Teaching Mathematics, 2015 by Marilyn Burns @ Mathsolutions.com. A perfect companion story to this problem (also written by Marilyn Burns) is the book titled, "Spaghetti and Meatballs for All."



A Row of Squares

If you line up one hundred squares in a row, what will the perimeter measure? (As in *A Row of Triangles*, you may think of this as a long banquet table made from individual square tables. How many people can be seated?)

With one square, the perimeter is 4 units.

With two squares, the perimeter is 6 units.

With three squares, the perimeter is 8 units.

Continue the pattern.

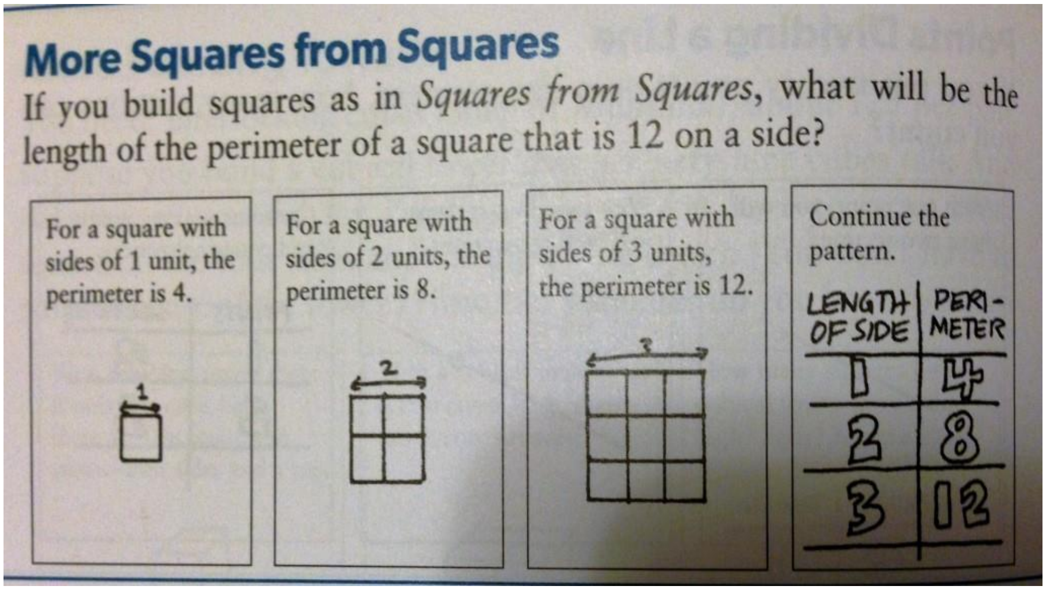
SQUARES	PERIMETER
1	4
2	6
3	8
4	

Row of Squares

Rule: +2

Formula: 2*n* + 2

Use the formula to name the perimeter if there are 7 square tables, 9? 10? Use square tiles or graph paper to check.



More Squares from Squares

If you build squares as in *Squares from Squares*, what will be the length of the perimeter of a square that is 12 on a side?

For a square with sides of 1 unit, the perimeter is 4.

For a square with sides of 2 units, the perimeter is 8.

For a square with sides of 3 units, the perimeter is 12.

Continue the pattern.

LENGTH OF SIDE	PERIMETER
1	4
2	8
3	12

Squares From Squares

Rule: +4

Formula: $4n$

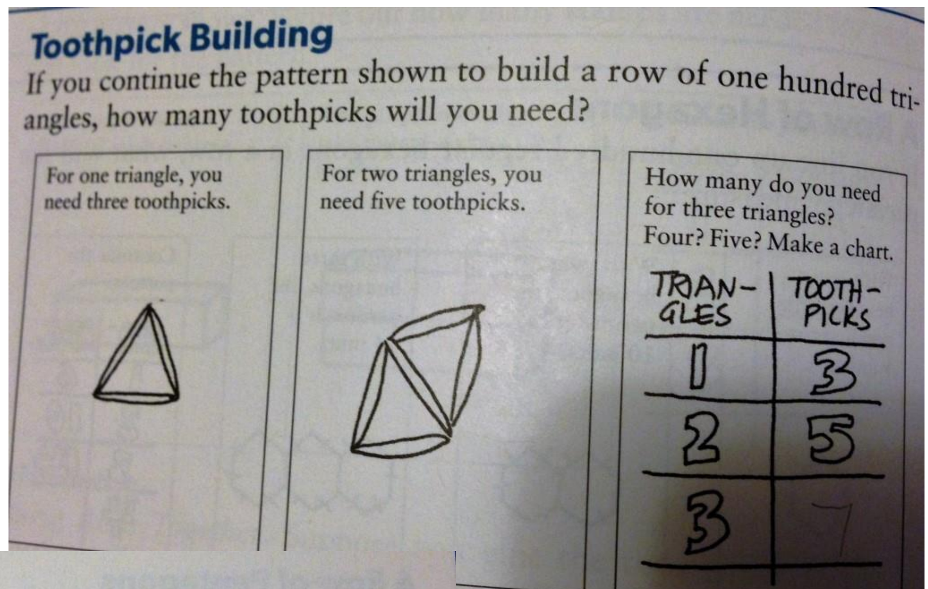
Predict the perimeter for a square with a length of 7 units, 9 units, 11 units, etc.

Toothpick Building

Rule: +2

Formula: $2n + 1$

Predict how many toothpicks will be needed to make 5 triangles, 10



Toothpick Building

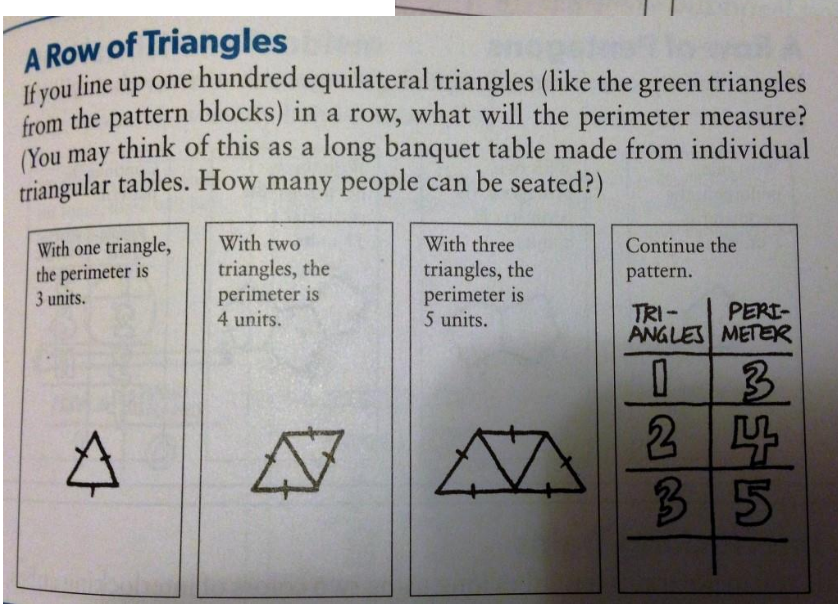
If you continue the pattern shown to build a row of one hundred triangles, how many toothpicks will you need?

For one triangle, you need three toothpicks.

For two triangles, you need five toothpicks.

How many do you need for three triangles? Four? Five? Make a chart.

TRIANGLES	TOOTHPICKS
1	3
2	5
3	7



A Row of Triangles

If you line up one hundred equilateral triangles (like the green triangles from the pattern blocks) in a row, what will the perimeter measure? (You may think of this as a long banquet table made from individual triangular tables. How many people can be seated?)

With one triangle, the perimeter is 3 units.

With two triangles, the perimeter is 4 units.

With three triangles, the perimeter is 5 units.

Continue the pattern.

TRIANGLES	PERIMETER
1	3
2	4
3	5

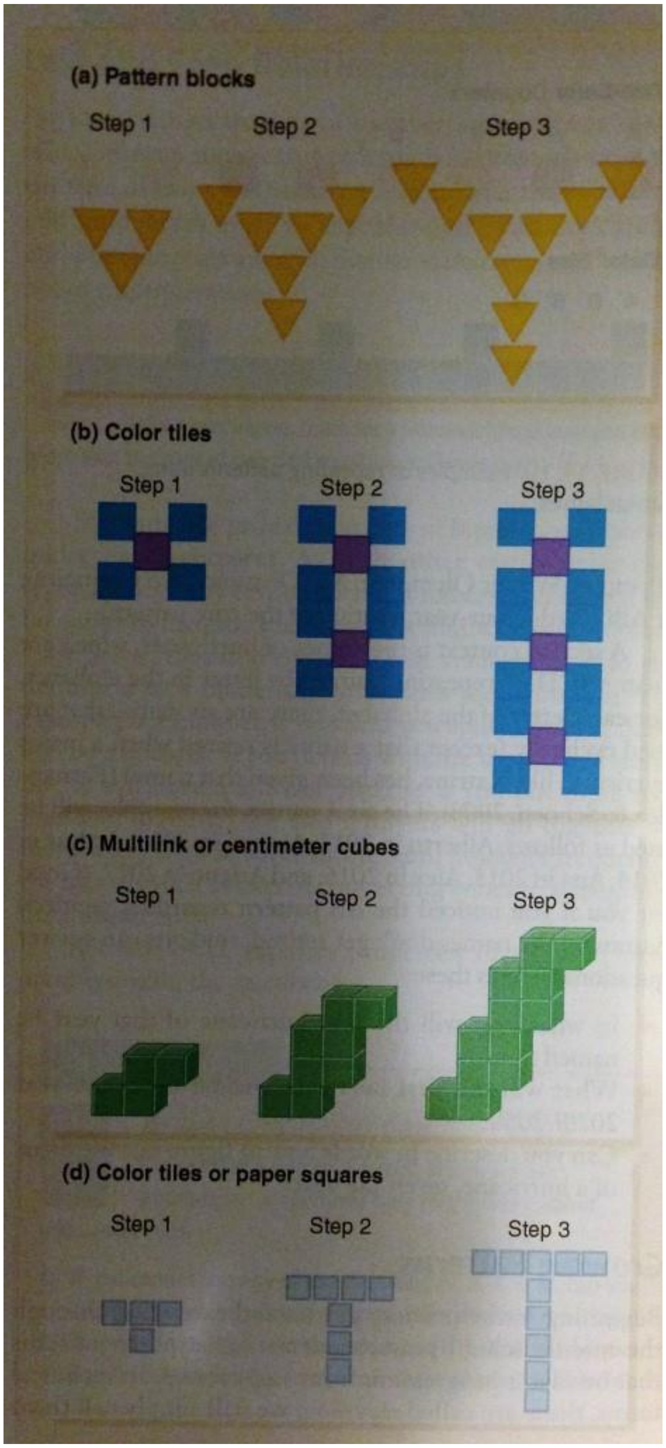
Row of Triangles

Rule: +1

Formula: $1n + 2$

What will the perimeter be if there are 6 triangles, 8? 10?

The illustrations below come from the book: Developing Algebraic and Geometric Reasoning (for Cameron University Math Spec. Program), by Pearson, 2013

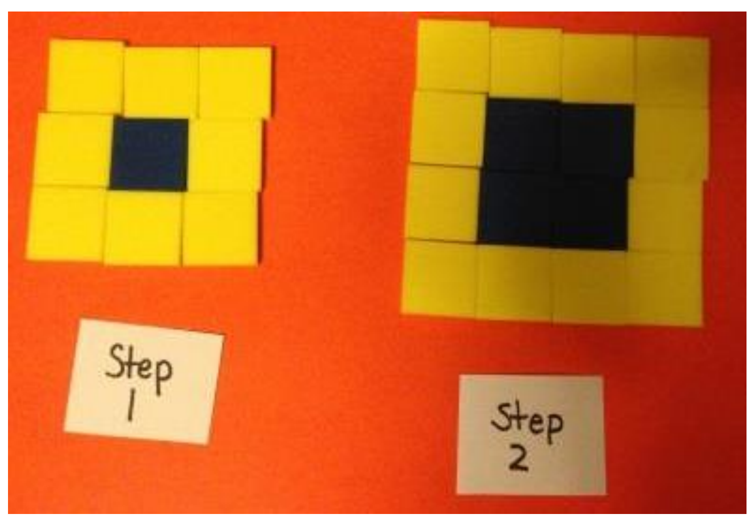


a) 3, 6, 9, ___, ___ Rule is +3; Formula is $3n$
b) 5, 10, 15, ___, ___ Rule is +5; Formula is $5n$
c) 4, 8, 12, ___, ____ Rule is +4; Formula is $4n$
d) 4, 7, 10, ___, ____ Rule is +3; Formula is $3n + 1$

Make a 2-column table for each picture. Label the left side with step #. Label the right side with # of objects. Continue the pattern. Predict the 10th one.

The Pool Problem

If your pool (blue tiles) forms a square, how many tiles are needed to surround it (yellow)? What formula could be used to determine the # of surrounding tiles without actually building the pool for a square pool with n number of tiles? First 2 steps are shown below.



Rule: +3 Formula: 3*n* -1	
Step #	Result
1	2
2	5
3	8
4	11
5	?

Graphing ordered pairs from data in function tables:

(1, 2)

(2, 5)

(5, 8)