

Twos: \* Double the number  
 \* Use commutative property  
 Ex:  $7 \times 2 \rightarrow 7$  groups of 2  
 Instead of counting by 2's seven times,  
 turn it around to:  $2 \times 7$   
 2 groups of 7  
 $7 + 7 = 14$

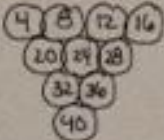
Threes: 

3	6	9
12	15	18
21	24	27
30		

 Make a 3 by 3 grid w/one extra box.  
 \* 1st row  $\rightarrow$  single digits  
 \* 2nd row  $\rightarrow$  teens  
 \* 3rd row  $\rightarrow$  twenties

1st column: All digits total 3  
 2nd column: Digits total 6  
 3rd column: Digits total 9

Three is an odd number.  
 Products using an odd factor alternate:  
 odd, even, odd, even ...

Fours: 
 I call this 4-3-2-1.  
 Looks like bowling pin set up or grapes cluster.

\* 1st row: less than 20  
 \* 2nd row: twenties  
 \* 3rd row: thirties

Four is an even number.  
 Therefore all products are even.

Even  $\times$  Even  $\# =$  Even product  
 Odd  $\times$  Even  $\# =$  Even product  
 Odd  $\times$  Odd  $\# =$  Odd product

Sixes: When multiplying 6 times an even \* factor < 10, notice the pattern:

$$6 \times 2 = \begin{array}{r} 12 \\ \hline \end{array}$$

$$6 \times 4 = \begin{array}{r} 24 \\ \hline \end{array}$$

$$6 \times 6 = \begin{array}{r} 36 \\ \hline \end{array}$$

$$6 \times 8 = \begin{array}{r} 48 \\ \hline \end{array}$$

The factor multiplied x 6 matches the one's place in the product.

Notice the ten's place in the product is half the one's place. Or it can be stated that the one's place is 2x the ten's place.

Eights:

8	16	24	32	40
48	56	64	72	80

Make a box 5 across and 2 down (shown).

- Skip count across the first 5 multiples of 8.
- Notice the one's place counts down by twos (8, 6, 4, 2, 0)
- Notice the ten's place increases by 1: 0, 1, 2, 3, 4
- To fill in the 2nd row, add 40 to the number above.
- The same pattern with one's and ten's exists in the 2nd row.
- All products are even.

Nines:

① Left side  
Count up from zero to 9

0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1
9	0

② Right side  
Count up from zero to 9 starting at bottom.

③ This shows the products.

④ Digits = 9 on all products.

⑤ Top half is reversed of bottom half.

## Area Model

*(65 × 3)*

	60 + 5	
3	180	15

$$\begin{array}{r} 180 \\ + 15 \\ \hline 195 \end{array}$$

*(12 × 13)*

	10 + 2	
10	100	20
3	30	6

$$130 + 26 = 156$$

*(OR)*

	10 + 2		
10	100	20	= 120
3	30	6	+ 36
			$\hline 156$

## Area Model

	60 + 5	
30	1800	150
4	240	20

$$2040 + 170$$

OR

1800	
240	
150	
20	
$\hline 2210$	

← Partial products

← answer

To do mentally → look for combinations of 100 or 10's.

	100 + 60 + 10	
	$\hline 2000$	
	+ 100	
	100 → (40 + 60)	
	10	
	$\hline 2210$	

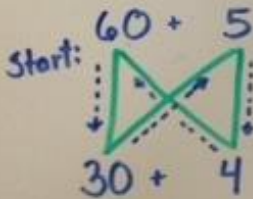
→ answer

# Bowtie / F.O.I.L.

$$65 \times 34$$

First, Outside, Inside, Last

- Make a large X
- Connect ends
- Break apart and multiply by following the lines. Then add.



$$\begin{array}{r} 60 \times 30 = 1800 \\ 30 \times 5 = 150 \\ 5 \times 4 = 20 \\ 4 \times 60 = 240 \\ \hline 2,210 \end{array}$$

$$65 \times 34 \\ (60+5) \times (30+4)$$

- First:  $60 \times 30 = 1800$
  - Outside:  $60 \times 4 = 240$
  - Inside:  $5 \times 30 = 150$
  - Last:  $5 \times 4 = 20$
- $$\begin{array}{r} 2,210 \end{array}$$

# Partial Products

$$\begin{array}{r} 65 \\ \times 3 \\ \hline 15 \rightarrow 3 \times 5 \\ 180 \rightarrow 3 \times 60 \\ \hline 195 \end{array}$$

This is related to distributive property:

$$\begin{array}{l} \underline{65} \times 3 \\ (60 \times 3) + (5 \times 3) \\ 180 + 15 = 195 \end{array}$$



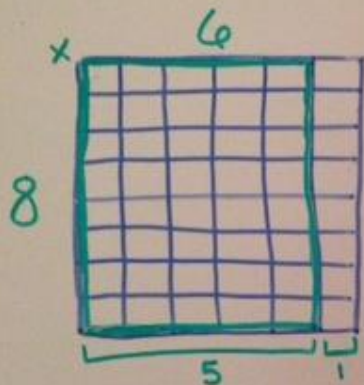
# Partial Products

$$\begin{array}{r} 65 \\ \times 34 \\ \hline 20 \rightarrow 4 \times 5 \\ 240 \rightarrow 4 \times 60 \\ 150 \rightarrow 30 \times 5 \\ 1800 \rightarrow 30 \times 60 \\ \hline 2,210 \end{array}$$

Distributive Property

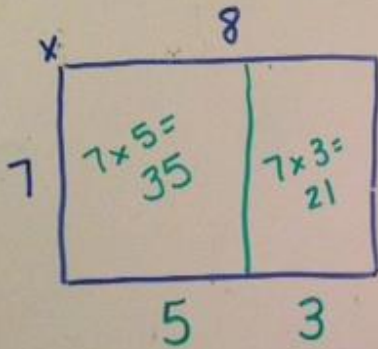
$$\begin{aligned} & 65 \times 34 \\ & (60+5) \times (30+4) \\ & (60 \times 30) + (60 \times 4) + \\ & (5 \times 30) + (5 \times 4) \\ & 1800 + 240 + \\ & 150 + 20 \\ & \hline 1950 + 260 = 2210 \end{aligned}$$

Problem:  $8 \times 6$



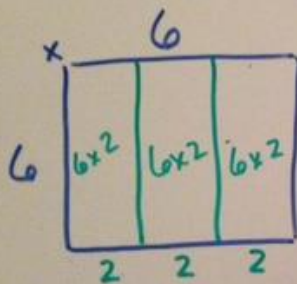
1. Break 1 of the factors into 2 parts - usually with a 2 or a 5.
2.  $8 \times 6$   
 $8 \times (5+1)$
3. Multiply  $8 \times 5 = 40$   
Multiply  $8 \times 1 = \frac{8}{48}$
4. Add these together.
5.  $8 \times 6 = 48$

Problem:  $7 \times 8$



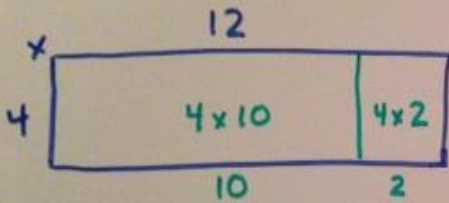
1. Break 1 of the factors into 2 parts - usually with a 2 or a 5.
2.  $7 \times 8$   
 $7 \times (5+3)$
3. Multiply  $7 \times 5 = 35$   
Multiply  $7 \times 3 = 21$   
 $\underline{56}$
4. Add these together.
5.  $7 \times 8 = 56$

Problem:  $6 \times 6$



1. Break 1 of the factors into 2 parts - usually with a 2 or a 5.
2.  $6 \times 6$   
 $6 \times (2+2+2)$
3. Multiply  $6 \times 2 = 12$   
Multiply  $6 \times 2 = 12$   
Multiply  $6 \times 2 = 12$   
 $\underline{36}$
4. Add these together.
5.  $6 \times 6 = 36$

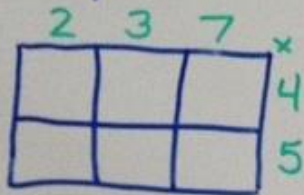
Problem:  $4 \times 12$



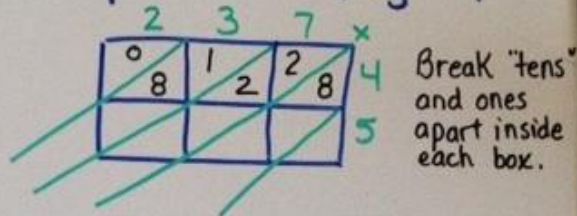
1. Break 1 of the factors into 2 parts - usually with a 2, 5 or 10.
2.  $4 \times 12$   
 $4 \times (10 + 2)$
3. Multiply  $4 \times 10 = 40$   
Multiply  $4 \times 2 = 8$   
 $\underline{+ 8}$   
48
4. Add these together.
5.  $4 \times 12 = 48$

Lattice for Multip.  $237 \times 45$

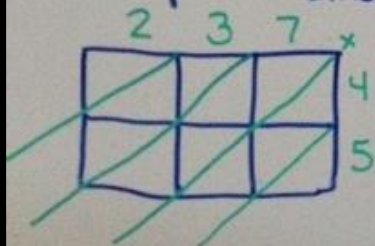
Step 1: Box



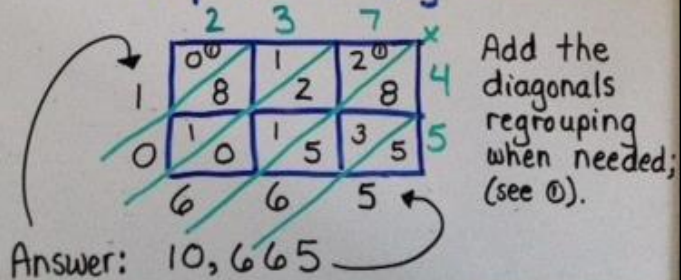
Step 3: Multiply top



Step 2: Diag. Lines



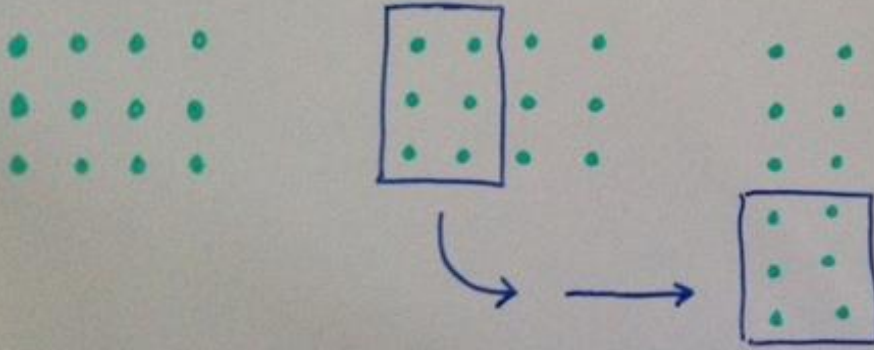
Step 4: Multiply bottom





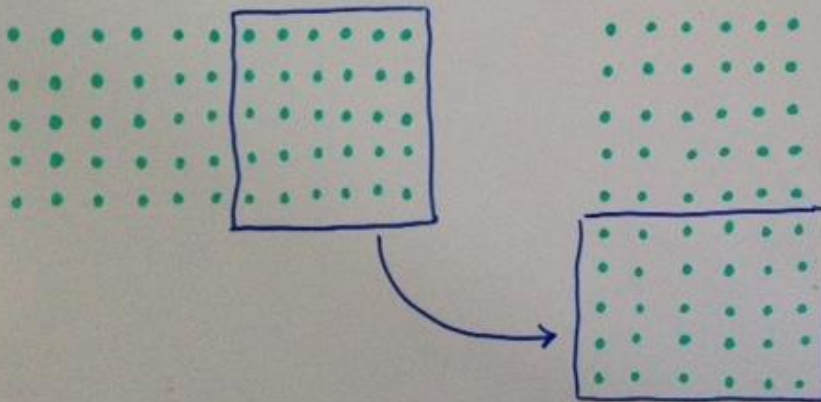
## Doubling and Halving,

$$3 \times 4 \rightarrow \begin{array}{l} \text{Double the 3} \\ \text{Halve the 4} \end{array} \rightarrow 6 \times 2 = 12$$



## Doubling and Halving,

$$5 \times 12 \rightarrow \begin{array}{l} \text{Double the 5} \\ \text{Halve the 12} \end{array} \rightarrow 10 \times 6 = 60$$





## Doubling and Halving,

$$\begin{array}{l} 25 \rightarrow \text{double the odd}^\# \rightarrow 50 \\ \times 12 \rightarrow \text{halve the even}^\# \rightarrow \begin{array}{r} \times 6 \\ \hline 300 \end{array} \end{array}$$

## Doubling and Halving,

$$\begin{array}{l} 15 \rightarrow \text{double} \rightarrow 30 \rightarrow \text{double} \rightarrow 60 \\ \times 24 \rightarrow \text{halve} \rightarrow \begin{array}{r} \times 12 \\ \hline \end{array} \rightarrow \text{halve} \rightarrow \begin{array}{r} \times 6 \\ \hline 360 \end{array} \end{array}$$

This one needed 2  
steps to solve mentally.

## How to double a #

17 doubled is     ?  
Double the ones = 14  
Double the tens = 20  
Add → 34

26 doubled is     ?  
Double the ones = 12  
Double the tens = 40  
Add → 52

## How to halve a number

Half of 64 is     ?  
Halve the ones = 2  
Halve the tens = 30  
Add → 32

## Multiply by 4

Double twice  
because  $4 = 2 \times 2$

$$5 \times 4 \rightarrow 5 \times 2 = 10 \\ 10 \times 2 = \textcircled{20}$$

$$12 \times 4 \rightarrow 12 \times 2 = 24 \\ 24 \times 2 = \textcircled{48}$$

$$31 \times 4 \rightarrow 31 \times 2 = 62 \\ 62 \times 2 = \textcircled{124}$$

## Multiply by 8

Double 3 times  
because  $8 = 2 \times 2 \times 2$

$$5 \times 8 \rightarrow 5 \times 2 = 10 \\ 10 \times 2 = 20 \\ 20 \times 2 = \textcircled{40}$$

$$12 \times 8 \rightarrow 12 \times 2 = 24 \\ 24 \times 2 = 48 \\ 48 \times 2 = \textcircled{96}$$

$$31 \times 8 \rightarrow 31 \times 2 = 62 \\ 62 \times 2 = 124 \\ 124 \times 2 = \textcircled{248}$$