

Partial Quotients Division Examples
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①

Problem: $252 \div 7$

1. Make a list of easy multiples of 7. Most children know multiples up to 5 quickly. Then use knowledge about multiples of 10 to write companion facts ($\times 10$, $\times 20$, etc.)
2. I can stop with 40×7 because it is greater than my dividend of 252.
3. Show the division problem in the long division format with a long vertical line.
4. Partial quotients will be written down the right side of the line.
5. Ask: How many 7's are in 252?
6. I want to estimate the quotient a part at a time. I can make several choices to get my first estimate, but using my easy list of multiples helps me think of ways multiplication is related to division. I will try 30 because 30×7 is the closest to my dividend of 252. $30 \times 7 = 210$.
7. As I get more experienced with learning multiplication facts and how to double and halve numbers, I can get closer to the actual dividend and reach my answer in less steps.
8. Subtract $252 - 210 = 42$.
9. Ask: How many 7s are in 42?
 - Using my easy multiples list, I can try 5. $5 \times 7 = 35$. (If I know that $6 \times 7 = 42$, then I can use 6 as my next partial quotient.)
 - Subtract $42 - 35 = 7$
10. Ask: How many 7s are in 7?
 - I can get one more group of 7. $1 \times 7 = 7$
 - Subtract $7 - 7 = 0$
11. Since there are 0 left over, I am done dividing.
12. Add all of the partial quotients:
 $30 + 5 + 1 = 36$
13. Answer: $254 \div 7 = 36$
14. I can check with multiplication:
 $36 \times 7 = 252$

$1 \times 7 = 7$	$10 \times 7 = 70$
$2 \times 7 = 14$	$20 \times 7 = 140$
$3 \times 7 = 21$	$30 \times 7 = 210$
$4 \times 7 = 28$	$40 \times 7 = 280$
$5 \times 7 = 35$	

$$\begin{array}{r}
 7 \overline{) 252} \\
 \underline{- 210} \\
 42 \\
 \underline{- 35} \\
 7 \\
 \underline{- 7} \\
 0
 \end{array}
 \begin{array}{l}
 30 \times 7 \\
 5 \times 7 \\
 1 \times 7 \\
 \hline
 36 \rightarrow \text{Add } \begin{array}{r} 30 \\ 5 \\ + 1 \\ \hline 36 \end{array}
 \end{array}$$

Check: 36×7

$$\begin{array}{r}
 \times \quad \begin{array}{|c|c|} \hline 30 & 6 \\ \hline \end{array} \\
 7 \overline{) \begin{array}{|c|c|} \hline 210 & 42 \\ \hline \end{array}}
 \end{array}$$

$$\begin{array}{r}
 210 \\
 + 42 \\
 \hline
 252 \quad \checkmark
 \end{array}$$

Problem: $1526 \div 23$

1. Make a list of easy multiples of 23. For double digit divisors, I will make a list using my knowledge of how to multiply $\times 2$ (double) and $\times 4$ (double twice) and $\times 8$ (double again). Then use knowledge about multiples of 10 to write companion facts ($\times 10$, $\times 20$, etc.)
2. I can stop with 80×23 because it is more than my dividend of 1526.
3. Show the division problem in the long division format with partial quotients listed along the right side.
4. Ask: How many 23s are in 1526?
5. I want to estimate the quotient a part at a time. I can make several choices to get my first estimate, but using my easy list of multiples helps me think of ways multiplication is related to division. I will try 40 because 40×23 is the closest to my dividend of 1526. $40 \times 23 = 920$.
6. As I get more experienced with learning multiplication facts and how to double and halve numbers, I can get closer to the actual dividend and reach my answer in less steps.
7. Subtract $1526 - 920 = 606$
8. Ask: How many 23s are in 606?
 - Using my multiples list, I can see $20 \times 23 = 460$ is the closest I can get to 606.
 - Subtract $606 - 460 = 146$
9. Ask: How many 23s are in 146?
 - Using my easy multiples list, I can see $4 \times 23 = 92$ is closest to 146.
 - Subtract: $146 - 92 = 54$
10. Ask: How many 23s are in 54?
 - Using my easy multiples list, I can see $2 \times 23 = 46$ is closest to 54.
 - Subtract: $54 - 46 = 8$
11. Since the amount left over is less than my divisor of 23, I am done. The remainder is 8.
12. Add all of the partial quotients:
 $40 + 20 + 4 + 2 = 66$ with a remainder of 8
13. Answer: $1526 \div 23 = 66 \text{ r}8$
14. I can check with multiplication:
 $(66 \times 23) + 8 = 1526$

$1 \times 23 = 23$	$10 \times 23 = 230$
$2 \times 23 = 46$	$20 \times 23 = 460$
$4 \times 23 = 92$	$40 \times 23 = 920$
$8 \times 23 = 184$	$80 \times 23 = 1840$

$ \begin{array}{r} 23 \overline{) 1526} \\ \underline{- 920} \\ 606 \\ \underline{- 460} \\ 146 \\ \underline{- 92} \\ 54 \\ \underline{- 46} \\ 8 \end{array} $	40×23 20×23 4×23 2×23 <hr/> $66 \rightarrow 40$ $+ 20$ $+ 4$ $+ 2$ <hr/> 66
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Check: $(66 \times 23) + 8$

$ \begin{array}{r} \begin{array}{ c c } \hline 20 & \begin{array}{ c c } \hline 60 & 6 \\ \hline \end{array} \\ \hline \end{array} \\ \begin{array}{ c c } \hline 3 & \begin{array}{ c c } \hline 180 & 18 \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} $	$1380 + 138 =$ <hr/> 1380 $+ 138$ $+ 8 \text{ remainder}$ <hr/> $1,526 \checkmark$
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Problem: $471 \div 9$

1. Make a list of easy multiples of 9. Most children know multiples up to 5 quickly. Then use knowledge about multiples of 10 to write companion facts ($\times 10$, $\times 20$, etc.)
2. I can stop with 50×9 because it is very close to my dividend of 471.
3. Show the division problem in the long division format with partial quotients listed along the right side.
4. Ask: How many 9s are in 471?
5. I want to estimate the quotient a part at a time. I can make several choices to get my first estimate, but using my easy list of multiples helps me think of ways multiplication is related to division. I will try 50 because 50×9 is the closest to my dividend of 471. $50 \times 9 = 450$.
6. As I get more experienced with learning multiplication facts and how to double and halve numbers, I can get closer to the actual dividend and reach my answer in less steps.
7. Subtract $471 - 450 = 21$
8. Ask: How many 9s are in 21?
 - Using my easy multiples list, I can see $2 \times 9 = 18$ is the closest I can get to 21.
 - Subtract $21 - 18 = 3$
9. Since the amount left over is less than my divisor of 9, I am done. The remainder is 3.
10. Add all of the partial quotients:
 $50 + 2 = 52$ with a remainder of 3
11. Answer: $471 \div 9 = 52 \text{ r } 3$.
12. I can check with multiplication:
 $(52 \times 9) + 3 = 471$

$1 \times 9 = 9$	$10 \times 9 = 90$
$2 \times 9 = 18$	$20 \times 9 = 180$
$3 \times 9 = 27$	$30 \times 9 = 270$
$4 \times 9 = 36$	$40 \times 9 = 360$
$5 \times 9 = 45$	$50 \times 9 = 450$

$$\begin{array}{r}
 9 \overline{) 471} \\
 \underline{- 450} \\
 21 \\
 \underline{- 18} \\
 3
 \end{array}
 \begin{array}{l}
 50 \times 9 \\
 2 \times 9 \\
 52 \rightarrow \text{Add} \\
 \begin{array}{r}
 50 \\
 + 2 \\
 \hline
 52
 \end{array}
 \end{array}$$

Remainder \rightarrow

Check: $(52 \times 9) + 3$

$$\begin{array}{r}
 \times 50 2 \\
 9 \overline{) 450 18}
 \end{array}$$

$$\begin{array}{r}
 450 \\
 + 18 \\
 + 3 \text{ remainder} \\
 \hline
 471 \quad \checkmark
 \end{array}$$

Other Examples of Partial Quotients Division:

$$\begin{array}{r} 6 \overline{) 98} \\ - 60 \\ \hline 38 \\ - 30 \\ \hline 8 \\ - 6 \\ \hline 2 \end{array}$$

Ans: 16 R 2

$1 \times 6 = 6$	$10 \times 6 = 60$
$2 \times 6 = 12$	$20 \times 6 = 120$
$3 \times 6 = 18$	
$4 \times 6 = 24$	
$5 \times 6 = 30$	

How many 6's in 98?

check: $(16 \times 6) + 2$

$$\begin{array}{r} \times \quad 10 \quad 6 \\ 6 \quad 60 \quad 36 \\ \hline 60 \\ + 36 \\ + 2R \\ \hline 98 \checkmark \end{array}$$

$$\begin{array}{r} 7 \overline{) 465} \\ - 350 \\ \hline 115 \\ - 70 \\ \hline 45 \\ - 35 \\ \hline 10 \\ - 7 \\ \hline 3 \end{array}$$

Ans: 66 R 3

$1 \times 7 = 7$	$10 \times 7 = 70$
$2 \times 7 = 14$	$20 \times 7 = 140$
$3 \times 7 = 21$	$30 \times 7 = 210$
$4 \times 7 = 28$	$40 \times 7 = 280$
$5 \times 7 = 35$	$50 \times 7 = 350$

How many 7's in 465?

check: $(66 \times 7) + 3$

$$\begin{array}{r} \times \quad 60 \quad 6 \\ 7 \quad 420 \quad 42 \\ \hline 420 \\ + 42 \\ + 3R \\ \hline 465 \checkmark \end{array}$$

$$\begin{array}{r} 8 \overline{) 1542} \\ - 800 \\ \hline 742 \\ - 400 \\ \hline 342 \\ - 320 \\ \hline 22 \\ - 16 \\ \hline 6 \end{array}$$

Ans: 192 R 6

$1 \times 8 = 8$	$10 \times 8 = 80$
$2 \times 8 = 16$	$20 \times 8 = 160$
$3 \times 8 = 24$	$30 \times 8 = 240$
$4 \times 8 = 32$	$40 \times 8 = 320$
$5 \times 8 = 40$	$50 \times 8 = 400$
	$100 \times 8 = 800$

How many 8's in 1542?

Check: $(192 \times 8) + 6$

$$\begin{array}{r} \times \quad 100 \quad 90 \quad 2 \\ 8 \quad 800 \quad 720 \quad 16 \\ \hline 800 \\ + 720 \\ + 16 \\ + 6R \\ \hline 1542 \checkmark \end{array}$$

$$\begin{array}{r} 9 \overline{) 154} \\ - 90 \\ \hline 64 \\ - 45 \\ \hline 19 \\ - 18 \\ \hline 1 \end{array}$$

Ans: 17 R 1

$1 \times 9 = 9$	$10 \times 9 = 90$
$2 \times 9 = 18$	$20 \times 9 = 180$
$3 \times 9 = 27$	
$4 \times 9 = 36$	
$5 \times 9 = 45$	

How many 9's in 154?

Check: $(17 \times 9) + 1$

$$\begin{array}{r} \times \quad 10 \quad 7 \\ 9 \quad 90 \quad 63 \\ \hline 90 \\ + 63 \\ + 1R \\ \hline 154 \checkmark \end{array}$$

Partial Quotients

How many — in — ?

4x =

1	4
2	8
3	12
4	16
5	20
10	40
20	80
30	120
100	400

$$\begin{array}{r}
 4 \overline{) 78} \\
 \underline{-40} \\
 38 \\
 \underline{-20} \\
 18 \\
 \underline{-16} \\
 2
 \end{array}
 \begin{array}{l}
 10 \\
 \\
 5 \\
 + 4 \\
 \hline
 19 R 2
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 97} \\
 \underline{-80} \\
 17 \\
 \underline{-16} \\
 1
 \end{array}
 \begin{array}{l}
 20 \\
 \\
 4 \\
 + 4 \\
 \hline
 24 R 1
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 539} \\
 \underline{-400} \\
 139 \\
 \underline{-120} \\
 19 \\
 \underline{-16} \\
 3
 \end{array}
 \begin{array}{l}
 100 \\
 \\
 30 \\
 \\
 4 \\
 + 4 \\
 \hline
 134 R 3
 \end{array}$$

$$\begin{array}{|c|c|}
 \hline
 10 & 9 \\
 \hline
 \end{array}
 \begin{array}{l}
 4 \overline{) 40 \quad 36} \\
 \hline
 \end{array}
 \begin{array}{l}
 = 76 \\
 + 2 \\
 \hline
 78
 \end{array}$$

$$\begin{array}{|c|c|}
 \hline
 20 & 4 \\
 \hline
 \end{array}
 \begin{array}{l}
 4 \overline{) 80 \quad 16} \\
 \hline
 \end{array}
 \begin{array}{l}
 96 \\
 + 1 \\
 \hline
 97
 \end{array}$$

$$\begin{array}{|c|c|c|}
 \hline
 100 & 30 & 4 \\
 \hline
 \end{array}
 \begin{array}{l}
 4 \overline{) 400 \quad 120 \quad 16} \\
 \hline
 \end{array}
 \begin{array}{l}
 400 \\
 120 \\
 16 \\
 \hline
 536 \\
 + 3 \\
 \hline
 539
 \end{array}$$

check w/ mult. using area model.

5x =

1	5
2	10
3	15
4	20
5	25
10	50
20	100
30	150
100	500

$$\begin{array}{r}
 5 \overline{) 63} \\
 \underline{-50} \\
 13 \\
 \underline{-10} \\
 3
 \end{array}
 \begin{array}{l}
 10 \\
 \\
 2 \\
 + 2 \\
 \hline
 12 R 3
 \end{array}$$

$$\begin{array}{|c|c|}
 \hline
 10 & 2 \\
 \hline
 \end{array}
 \begin{array}{l}
 5 \overline{) 50 \quad 10} \\
 \hline
 \end{array}
 \begin{array}{l}
 = 60 \\
 + 3 \\
 \hline
 63
 \end{array}$$

$$\begin{array}{r}
 5 \overline{) 428} \\
 \underline{-150} \\
 278 \\
 \underline{-150} \\
 128 \\
 \underline{-100} \\
 28 \\
 \underline{-25} \\
 3
 \end{array}
 \begin{array}{l}
 30 \\
 \\
 30 \\
 \\
 20 \\
 \\
 5 \\
 + 5 \\
 \hline
 85 R 3
 \end{array}$$

$$\begin{array}{|c|c|}
 \hline
 80 & 5 \\
 \hline
 \end{array}
 \begin{array}{l}
 5 \overline{) 400 \quad 25} \\
 \hline
 \end{array}
 \begin{array}{l}
 425 \\
 + 3 \\
 \hline
 428
 \end{array}$$

$$\begin{array}{r}
 5 \overline{) 832} \\
 \underline{-500} \\
 332 \\
 \underline{-250} \\
 82 \\
 \underline{-50} \\
 32 \\
 \underline{-30} \\
 2
 \end{array}
 \begin{array}{l}
 100 \\
 \\
 50 \\
 \\
 10 \\
 \\
 6 \\
 + 6 \\
 \hline
 166 R 2
 \end{array}$$

$$\begin{array}{|c|c|c|}
 \hline
 100 & 60 & 6 \\
 \hline
 \end{array}
 \begin{array}{l}
 5 \overline{) 500 \quad 300 \quad 30} \\
 \hline
 \end{array}
 \begin{array}{l}
 = 830 \\
 + 2 \\
 \hline
 832
 \end{array}$$

- Divide
- Multiply
- Subtract

Partial Quotients

How many — in —?

6x =

1	6
2	12
3	18
4	24
5	30
10	60
20	120
30	180
100	600

$$\begin{array}{r} 6 \overline{) 75} \\ \underline{-60} 10 \\ 15 \\ \underline{-12} +2 \\ 3 12 R3 \end{array}$$

$$\begin{array}{r} 10 + 2 \\ 6 \overline{) 60 12} = 72 \\ + 3 \\ \hline 75 \end{array}$$

$$\begin{array}{r} 6 \overline{) 98} \\ \underline{-60} 10 \\ 38 \\ \underline{-30} 5 \\ 8 \\ \underline{-6} +1 \\ 2 16 R2 \end{array}$$

$$\begin{array}{r} 10 + 6 \\ 6 \overline{) 60 36} = 96 \\ + 2 \\ \hline 98 \end{array}$$

$$\begin{array}{r} 6 \overline{) 321} \\ \underline{-300} 50 \\ 21 \\ \underline{-18} +3 \\ 3 53 R3 \end{array}$$

$$\begin{array}{r} 50 + 3 \\ 6 \overline{) 300 18} = 318 \\ + 3 \\ \hline 321 \end{array}$$

Check w/ multip. using area model.

7x =

1	7
2	14
3	21
4	28
5	35
10	70
20	140
30	210
100	700

$$\begin{array}{r} 7 \overline{) 87} \\ \underline{-70} 10 \\ 17 \\ \underline{-14} +2 \\ 3 12 R3 \end{array}$$

$$\begin{array}{r} 10 + 2 \\ 7 \overline{) 70 14} = 84 \\ + 3 \\ \hline 87 \end{array}$$

$$\begin{array}{r} 7 \overline{) 156} \\ \underline{-70} 10 \\ 86 \\ \underline{-70} 10 \\ 16 \\ \underline{-14} +2 \\ 2 22 R2 \end{array}$$

$$\begin{array}{r} 20 + 2 \\ 7 \overline{) 140 14} = 154 \\ + 2 \\ \hline 156 \end{array}$$

$$\begin{array}{r} 7 \overline{) 465} \\ \underline{-350} 50 \\ 115 \\ \underline{-70} 10 \\ 45 \\ \underline{-35} 5 \\ 10 \\ \underline{-7} +1 \\ 3 66 R3 \end{array}$$

$$\begin{array}{r} 60 + 6 \\ 7 \overline{) 420 42} = 462 \\ + 3 \\ \hline 465 \end{array}$$

- Divide
- Multiply
- Subtract

Partial Quotients

How many — in — ?

8x =

1	8
2	16
3	24
4	32
5	40
10	80
20	160
30	240
100	800

$$\begin{array}{r} 8 \overline{) 98} \\ \underline{-80} 10 \\ 18 \\ \underline{-16} +2 \\ 2 12 \text{ R } 2 \end{array}$$

$$\begin{array}{r} 10 + 2 \\ 8 \overline{) 80 16} = 96 \\ \phantom{8 \overline{) 80 16}} + 2 \\ \phantom{8 \overline{) 80 16}} \hline 98 \end{array}$$

$$\begin{array}{r} 8 \overline{) 281} \\ \underline{-240} 30 \\ 41 \\ \underline{-40} +5 \\ 1 35 \text{ R } 1 \end{array}$$

$$\begin{array}{r} 30 + 5 \\ 8 \overline{) 240 40} = 280 \\ \phantom{8 \overline{) 240 40}} + 1 \\ \phantom{8 \overline{) 240 40}} \hline 281 \end{array}$$

$$\begin{array}{r} 8 \overline{) 1542} \\ \underline{-800} 100 \\ 742 \\ \underline{-400} 50 \\ 342 \\ \underline{-320} 40 \\ 22 \\ \underline{16} 2 \\ 6 192 \text{ R } 6 \end{array}$$

$$\begin{array}{r} 100 + 90 + 2 \\ 8 \overline{) 800 720 16} \\ \phantom{8 \overline{) 800 720 16}} 800 \\ \phantom{8 \overline{) 800 720 16}} \underline{-720} 16 \\ \phantom{8 \overline{) 800 720 16}} 1536 \\ \phantom{8 \overline{) 800 720 16}} + 6 \\ \phantom{8 \overline{) 800 720 16}} \hline 1542 \end{array}$$

Check w/multipl. using area model.

9x =

1	9
2	18
3	27
4	36
5	45
10	90
20	180
30	270
100	900

$$\begin{array}{r} 9 \overline{) 154} \\ \underline{-90} 10 \\ 64 \\ \underline{-45} 5 \\ 19 \\ \underline{-18} +2 \\ 1 17 \text{ R } 1 \end{array}$$

$$\begin{array}{r} 10 + 7 \\ 9 \overline{) 90 63} = 90 \\ \phantom{9 \overline{) 90 63}} + 63 \\ \phantom{9 \overline{) 90 63}} \hline 153 \\ \phantom{9 \overline{) 90 63}} + 1 \\ \phantom{9 \overline{) 90 63}} \hline 154 \end{array}$$

$$\begin{array}{r} 9 \overline{) 1628} \\ \underline{-270} 30 \\ 358 \\ \underline{-270} 30 \\ 88 \\ \underline{-45} 5 \\ 43 \\ \underline{-36} +4 \\ 7 69 \text{ R } 7 \end{array}$$

$$\begin{array}{r} 60 + 9 \\ 9 \overline{) 540 81} = 540 \\ \phantom{9 \overline{) 540 81}} + 81 \\ \phantom{9 \overline{) 540 81}} \hline 621 \\ \phantom{9 \overline{) 540 81}} + 7 \\ \phantom{9 \overline{) 540 81}} \hline 628 \end{array}$$

$$\begin{array}{r} 9 \overline{) 1428} \\ \underline{-900} 100 \\ 528 \\ \underline{-450} 50 \\ 78 \\ \underline{-72} +8 \\ 6 158 \text{ R } 6 \end{array}$$

$$\begin{array}{r} 100 + 50 + 8 \\ 9 \overline{) 900 450 72} = 900 \\ \phantom{9 \overline{) 900 450 72}} + 450 \\ \phantom{9 \overline{) 900 450 72}} 72 \\ \phantom{9 \overline{) 900 450 72}} \hline 1422 \\ \phantom{9 \overline{) 900 450 72}} + 6 \\ \phantom{9 \overline{) 900 450 72}} \hline 1428 \end{array}$$

- Divide
- Multiply
- Subtract